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CORRECTION FACTORS IN THE DETERMINATION OF MEAN VELOCITY OF OVERLAND FLOW

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ABSTRACT

The velocity of overland flow has been conventionally measured using tracers, but it is difficult to measure the mean flow velocity directly because the centroid of the tracer plume is not easily identified. Consequently, previous investigators have measured the velocity of the leading edge of the plume and multiplied it by a correction factor α to obtain an estimate of mean velocity. An alternative method is to measure the velocity of the peak concentration in the tracer plume and multiply this velocity by another correction factor β to estimate mean velocity. To investigate the controls of α and β and develop predictive models for these correction factors, 40 experiments were performed in a flume with a mobile sand bed. Multiple regression analyses reveal that both α and β vary inversely with slope and directly with Reynolds number. The derived regression equations may be used to calculate the mean velocity of other shallow overland flows, at least within the range of slope and Reynolds number for which the equations were developed. In the experiments, slope ranged from 2·7° to 10° and Reynolds number from 1900 to 12 600.

KEY WORDS overland flow; flow velocity; flow tracing; hillslope processes

INTRODUCTION

In the investigation of overland flow hydraulics, mean flow velocity $\bar{U}$ must be either calculated from other hydraulic variables or measured directly. Where flow velocity is measured, it has usually been done by means of visually tracing a cloud of dye (e.g. Horton *et al.*, 1934; Emmett, 1970; Abrahams *et al.*, 1986) or electronically monitoring the passage of a plume of dye using a fluorometer (Gilley *et al.*, 1990) or a plume of salt using a conductivity meter (King and Norton, 1992; Luk and Merz, 1992; Abrahams and Atkinson, 1993). The time from the injection of the tracer to the arrival of the leading edge T_e, the peak T_p, or the centroid T_c of the tracer concentration distribution at a sampling station downslope is measured and then divided by the travel distance to give the leading edge velocity U_e, the peak concentration velocity U_p, or the centroid velocity U_c, respectively. According to the theory of advection and diffusion in open channel flow, $\bar{U} = U_c$ (Elder, 1959). In addition, for a finite travel distance, the tracer concentration distributed with respect to time is right-skewed (Taylor, 1954), so $U_p > \bar{U}$. However, when visual tracing is employed, the centroid of the tracer plume is impossible to identify. When electronic tracing is used, the centroid can be identified from the tracer concentration distribution, but not as easily as can the leading edge or peak of the distribution. For these reasons, it has become common practice in studies of overland flow to measure U_e and multiply this velocity by a correction factor to obtain an estimate of $\bar{U}$. Several studies have sought to determine the value of this correction factor $\bar{U}/U_e$, hereafter denoted by α.

For laminar flow over a smooth bed, Horton *et al.* (1934) showed theoretically that the ratio of surface velocity to $\bar{U}$ is 0·67. Many investigators, including that of Horton *et al.*, have assumed that U_e is equivalent

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to surface velocity and subsequently that $\alpha = 0{\cdot}67$ for laminar flow. In a comprehensive set of laboratory experiments, Emmett (1970) found that α generally fell between 0·5 to 0·6 for laminar flow, increased with Reynolds number for transitional flow, and was close to 0·8 for turbulent flow. In addition, in field experiments he reported that α was about 0·4 to 0·5 for laminar flow. Likewise, in field experiments Luk and Merz (1992) obtained a mean value for α of 0·52 for laminar flow, while in laboratory experiments they obtained a mean value for α of 0·75 for transitional and turbulent flows. These results are consistent with experiments by Phelps (1975) which showed that velocity profiles in laminar flow were steeper over a rough bed than a smooth one, thereby implying that $\alpha < 0{\cdot}67$ for laminar flow over a rough bed. In contrast to the aforementioned studies which suggest that α is dependent on Reynolds number, King and Norton's (1992) experiments in an artificial laboratory rill indicated that for flows ranging from laminar to turbulent, α was unrelated to discharge (i.e. Reynolds number) but inversely related to slope. Thus although there is the suggestion that α varies with Reynolds number and/or slope, the evidence is meagre and ambiguous. Moreover, there exists no reliable method of predicting α for a given overland flow.

An alternative to estimating $\bar{U}$ from U_e is to estimate $\bar{U}$ from U_p by multiplying U_p by β where $\beta = \bar{U}/U_p$. This procedure may be used in addition to or instead of multiplying U_e by α. Hitherto, no one has investigated the value of β for overland flow.

The objectives of this paper are to investigate the controls of α and β and to develop predictive models for these correction factors. The investigation is based on a set of 40 experiments conducted in a laboratory flume with a mobile sand bed. Simulated rainfall was not employed in these experiments.

EXPERIMENT SET-UP

The flume was 520 cm long and 40 cm wide with smooth Plexiglas walls (Figure 1). It consisted of two parts: a lower part, 360 cm long, with a mobile sand-covered bed and an upper, steeper part, 160 cm long, with a smooth aluminium bed. For the experiments the lower part of the flume was inclined at three slopes θ: 2·7°, 5·5° and 10·0°. The sand, which was a well-sorted silica testing sand (ASTM C-190) with a median diameter of 0·74 mm, was supplied by a continuously adjustable sediment feed system to the upper part of the flume and was trapped at the lower end of the flume in two containers. Water entered the upper end of the flume by overflowing from a head tank. The unit water discharge, which ranged from 3·04 to 33·41 cm^2 s^{-1}, was controlled by a gate valve and measured with an OMEGA FP-5300 paddlewheel flow meter. For each experiment, the sediment feed rate was adjusted to the water discharge so that the bed was experiencing no perceptible scour or deposition.

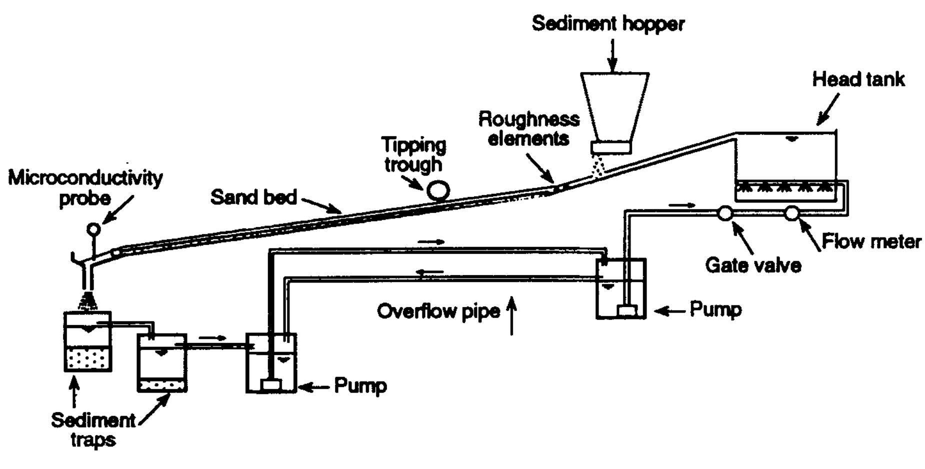

Figure 1. Sketch of the flume

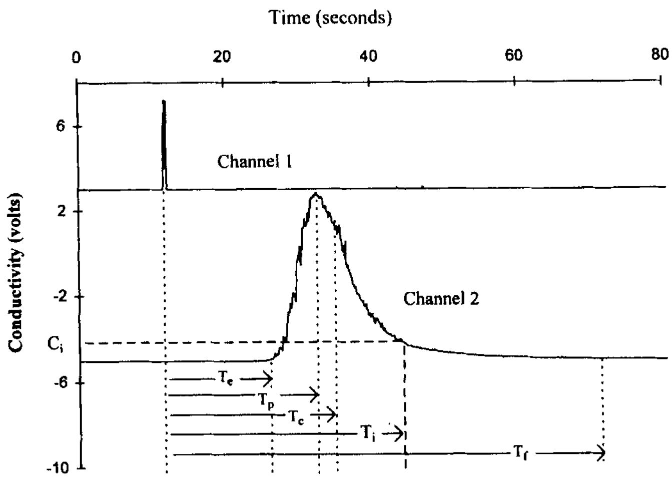

Figure 2. Time distribution of conductivity

Flow velocities were determined from the movement of a saline plume down the flume. A small quantity (60 to 100 cm^3) of highly saline water was placed in a trough located 260 cm from the lower end of the flume. The trough was the same width as the flume to avoid lateral diffusion of the salt solution. The tipping of the trough completed an electrical circuit, which sent a voltage signal to one channel of a data acquisition system. This signal marked the time the saline solution entered the flow. The flow covered the entire bed but contained threads of varying width, depth and position. In order to measure U_e, U_p and $\bar{U}$ for the entire flow, a 20 cm long funnel was constructed at the lower end of the flume to concentrate the flow, and a microconductivity probe was placed in the neck of the funnel. Measurements showed that the increase in slope and depth of flow through the funnel more or less compensated for its tendency to throttle the flow, so that flow velocity remained much the same through the funnel as down the flume. The probe produced a voltage signal in response to the change in conductivity of the water as the salt-rich plume passed. This signal was sent to a second channel of the data acquisition system. The conductivity was measured at a rate of 25 Hz over an interval of 80 s, which was adequate to record the entire plume. The recorded data were then processed by a computer program to yield T_e, T_p and T_c (Figure 2), where T_e is the time interval from the tipping of the trough to the initial rise in conductivity above the base value, T_p is the time interval from the tipping of the trough to the peak value of conductivity, and T_c is calculated by the moment equation:

$$T_c = \frac{\sum_{T_e}^{T_f} C_i T_i}{\sum_{T_e}^{T_f} C_i} \tag{1}$$

In this equation, T_f refers to the ending time of the concentration distribution, which is read from the conductivity distribution graph; C_i is the conductivity, corresponding to salinity, sensed by the probe; and T_i is the time when C_i is measured. The results of the experiments are listed in Table I.

CORRECTION FACTOR α

Previous studies have suggested that α may vary with slope and/or Reynolds number. To investigate this proposition, $S = \sin\theta$ and Reynolds number $Re = 4d\bar{U}/\nu$, where d is the mean flow depth and ν is the

Table I. Experimental data

Experiment number	Flume slope (sine)	Unit water discharge ($cm^2 s^{-1}$)	Edge velocity ($cm s^{-1}$)	Peak concentration velocity ($cm s^{-1}$)	Mean velocity ($cm s^{-1}$)	Reynolds number	Flow depth (cm)
1	0·047	14·57	47·9	34·0	30·6	5474	0·48
2	0·047	11·67	47·0	32·4	27·7	4394	0·42
3	0·047	5·52	35·0	23·8	19·3	2083	0·29
4	0·047	5·14	35·5	23·1	18·9	1940	0·27
5	0·047	9·98	41·7	30·3	25·3	3775	0·39
6	0·047	8·83	40·7	26·8	24·4	3347	0·36
7	0·047	11·80	47·0	30·6	28·0	4471	0·42
8	0·047	18·39	54·3	40·2	34·9	6969	0·53
9	0·047	21·61	61·4	41·4	37·8	8189	0·57
10	0·047	24·67	65·4	43·8	40·0	9348	0·62
11	0·047	26·87	63·6	45·2	41·9	10 185	0·64
12	0·047	29·49	63·6	48·3	42·2	11 202	0·70
13	0·047	30·39	63·6	47·9	43·7	11 543	0·70
14	0·047	31·98	65·4	50·0	44·1	12 122	0·73
15	0·047	31·83	71·4	52·2	47·3	12 035	0·67
16	0·047	6·77	34·3	23·8	20·0	2559	0·34
17	0·047	6·56	36·5	25·1	20·9	2486	0·31
18	0·096	13·64	63·6	41·7	32·3	4849	0·42
19	0·096	10·44	59·3	36·6	29·0	3719	0·36
20	0·096	7·81	53·4	32·8	24·2	2787	0·32
21	0·096	7·22	52·6	29·3	22·6	2579	0·32
22	0·096	5·31	50·4	27·0	21·1	1902	0·25
23	0·096	22·55	75·3	41·7	36·2	8069	0·62
24	0·096	19·02	68·6	46·4	38·6	6820	0·49
25	0·096	22·08	76·1	47·0	39·9	7918	0·55
26	0·096	23·97	85·4	53·8	44·9	8596	0·53
27	0·096	25·67	82·4	46·9	40·7	9207	0·63
28	0·096	35·07	83·3	60·3	47·0	12 606	0·75
29	0·096	33·12	87·5	53·0	42·7	11 903	0·78
30	0·096	31·54	85·4	52·2	43·4	11 336	0·73
31	0·174	7·19	61·9	34·5	24·8	2690	0·29
32	0·174	6·47	62·5	39·3	24·5	2424	0·26
33	0·174	12·85	92·1	52·2	40·4	4828	0·32
34	0·174	12·46	101·5	58·8	40·8	4680	0·31
35	0·174	15·28	95·9	62·5	44·8	5792	0·34
36	0·174	14·45	116·7	65·4	47·3	5487	0·31
37	0·174	8·45	56·0	33·5	24·1	3218	0·35
38	0·174	7·93	62·0	39·3	27·2	3027	0·29
39	0·174	10·66	66·7	39·8	29·1	4067	0·37
40	0·174	10·65	81·4	43·8	33·1	4061	0·32

kinematic viscosity of the fluid, were calculated for each of the 40 experiments (Table I). A stepwise multiple regression was then performed with α as the dependent variable and $\log S$ and $\log Re$ as the independent variables. Both independent variables entered the regression, giving the result:

$$\alpha = -0{\cdot}251 - 0{\cdot}327 \log S + 0{\cdot}114 \log Re \tag{2}$$

with a multiple coefficient of determination $R^2 = 0{\cdot}896$ (Table II). Equation 2 is represented by the plots in Figure 3.

Table II. Stepwise regression results

Dependent variable	Regression constant	Regression coefficients / Standardized regression coefficients: log S	log Re	Coefficient of determination	Standard error of estimate	Sample size
α	−0·251	−0·327*	0·114*	0·896	0·029	40
		−0·833	0·322			
β	0·154	−0·257*	0·102*	0·839	0·030	40
		−0·788	0·347			

* Regression coefficient is significantly different from zero at the 0·05 level

CORRECTION FACTOR β

Inasmuch as α varies with both S and Re, it seems likely that β will do likewise. To examine this proposition a stepwise regression was conducted with β as the dependent variable and log S and log Re as the independent variables. As was the case with α, both independent variables entered the regression, yielding:

$$\beta = 0{\cdot}154 - 0{\cdot}257 \log S + 0{\cdot}102 \log Re \tag{3}$$

with $R^2 = 0{\cdot}839$ (Table II). Equation 3 is represented by the plots in Figure 4.

DISCUSSION

Equations 2 and 3 show that both α and β vary inversely with S and directly with Re. Perhaps surprisingly, given the emphasis of the literature on the effect of Re, S has a stronger influence on both α and β than

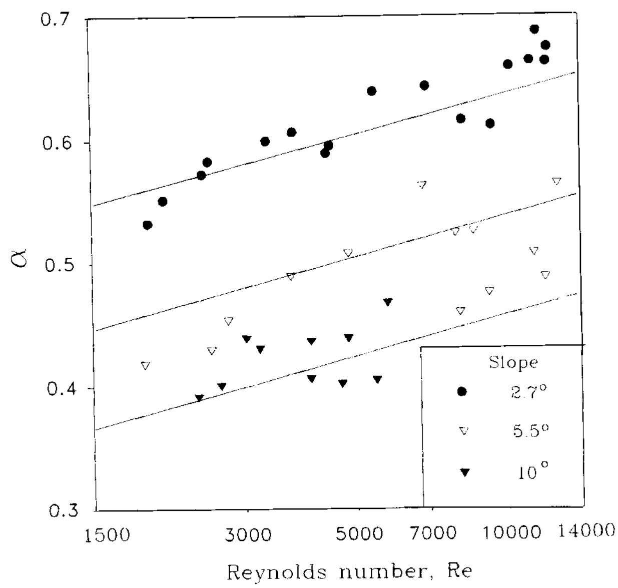

Figure 3. Variation of α with slope and Reynolds number. The lines plotted from Equation 2 are:
$\alpha = 0{\cdot}183 + 0{\cdot}114 \log Re$ for $\theta = 2{\cdot}7°$
$\alpha = 0{\cdot}082 + 0{\cdot}114 \log Re$ for $\theta = 5{\cdot}5°$
$\alpha = -0{\cdot}002 + 0{\cdot}114 \log Re$ for $\theta = 10°$

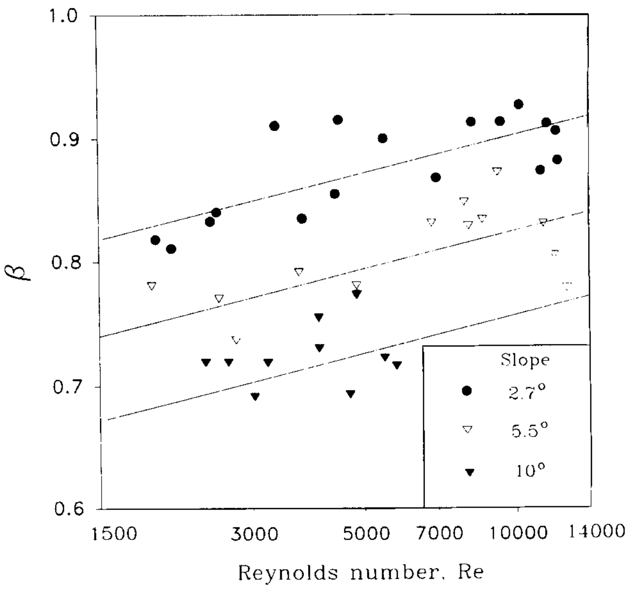

Figure 4. Variation of β with slope and Reynolds number. The lines plotted from Equation 3 are:
$\beta = 0{\cdot}495 + 0{\cdot}102 \log Re$ for $\theta = 2{\cdot}7°$
$\beta = 0{\cdot}416 + 0{\cdot}102 \log Re$ for $\theta = 5{\cdot}5°$
$\beta = 0{\cdot}349 + 0{\cdot}102 \log Re$ for $\theta = 10°$

does *Re*. This is indicated by the standardized regression coefficients in Table II. Much of the scatter in the data in Figures 3 and 4 may be ascribed to the mobile nature of the bed, which permitted the flow to concentrate into threads to different degrees in different experiments. Even so, the R^2 values associated with Equations 2 and 3 are sufficiently high that these equations can be used with a high degree of confidence to estimate α and β and, hence, calculate $\bar{U}$ from a knowledge of U_e or U_p.

Care should be taken, however, not to apply these equations beyond the ranges of θ and *Re* for which they have been developed—that is, $2{\cdot}7° \leqslant \theta \leqslant 10°$ and $1900 \leqslant Re \leqslant 12\,600$. It is noteworthy that most of the experiments have values of *Re* indicative of transitional flow (i.e. $2000 \leqslant Re \leqslant 8000$ approximately). Thus the direct relation between α and *Re* signified by Equation 2 may simply reflect an increase in α with *Re* in the transitional flow regime from a relatively low constant value for laminar flow to a relatively high constant value for turbulent flow (Emmett, 1970). If this is the case, the direct relation between α and *Re* evident in the present data will not apply to either lower or higher values of *Re* than those observed. Additional experiments with flows having lower and higher values of *Re* are required to resolve this question.

Consistent with the findings of Emmett (1970) and Luk and Merz (1992), the α values recorded for the present experiments in the laminar flow regime are smaller than the theoretical value of 0·67 derived by Horton *et al.* (1934) for the ratio of mean velocity to surface velocity for laminar flow over a smooth bed. A possible explanation for this finding is that the present experiments, like those of Emmett (1970) and Luk and Merz (1992), were conducted on rough beds. Velocity profiles on rough beds are steeper than those on smooth beds (Phelps, 1975), causing α to be less than 0·67.

CONCLUSION

The velocity of overland flow has conventionally been measured using tracers, but it is difficult to measure the mean flow velocity directly because the centroid of the tracer plume is not easily identified. Consequently, previous investigators have measured the velocity of the leading edge of the tracer plume and multiplied it by a correction factor α to obtain an estimate of mean velocity. An alternative method of estimating mean

velocity is to measure the velocity of the peak concentration in the tracer plume and multiply this velocity by another correction factor β. In this study, 40 experiments were performed without rainfall in a flume with a mobile sand bed to investigate the controls of α and β and to develop predictive models for these correction factors. Salt was used as the tracer in these experiments, and its passage was recorded by a microconductivity probe. Multiple regression analyses reveal that both α and β vary inversely with slope and directly with Reynolds number. The derived regression equations may be used to calculate mean velocity for other shallow overland flows, but not beyond the range of slope and Reynolds numbers for which the equations were developed. In applying these equations, it is necessary to compute Reynolds number from unit water discharge obtained by dividing water discharge by flow width rather than by multiplying d times $\bar{U}$, as $\bar{U}$ is unknown. Finally, the finding in the present experiments that $\alpha < 0{\cdot}67$ for laminar flow implies that both α and β are inversely related to bed roughness. Further experiments using different sand sizes are required to explore this implication.

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